Question	Scheme	Marks
1(a)	$(3,-1)$	B1B1
		(2)
(b)	$\left(-\frac{7}{2},5\right)$	B1B1
		(2)
		(4 marks)

Notes:

(a)

B1 Correct x coordinate

B1 Correct y coordinate

(b)

B1 Correct x coordinate

B1 Correct y coordinate

Question	Scheme	Marks
2(a)	$\cos(A+A)=\cos^2 A-\sin^2 A=(1-\sin^2 A)-\sin^2 A$	M1
	$\cos 2A = 1-2\sin^2 A$ *	A1*
		(2)
(b)	$\int_0^{\frac{\pi}{4}} 3\sin^2 2x\,\mathrm{d}x = 3\int_0^{\frac{\pi}{4}}\left(\frac{1}{2}-\frac{\cos 4x}{2}\right)\mathrm{d}x$	M1
	$\left[\frac{3}{2}x-\frac{3}{8}\sin 4x\right]_0^{\frac{\pi}{4}}=\left(\frac{3}{2}\times\frac{\pi}{4}-0\right)-0$	M1
	$=\frac{3\pi}{8}$	A1
		(3)
		(5 marks)

Notes:

(a)

M1 Attempts the compound angle formulae and applies $\cos^2 A = 1-\sin^2 A$

A1* Achieves the given answer with no errors or missing brackets.

(b)

M1 Attempts to replace $\sin^2 2x$ with $\left(\frac{1}{2}-\frac{1}{2}\cos 4x\right)$. As a minimum look for an expression of the form $\int a \pm b\cos 4x\ \mathrm{d}x$.

M1 Integrates $\ldots\cos 4x \to \ldots\sin 4x$ **and** substitutes in the limits of $\frac{\pi}{4}$ and 0 into their changed function (subtracts either way round)

A1 $\frac{3\pi}{8}$ cao

Note an answer with no working in (b) is 0 marks

Question	Scheme	Marks
3(a)	$a+60\mathrm{e}^{-0.05\times0}=2(100+80\mathrm{e}^{0.05\times0})$	M1
	$a=200+160-60=300$	A1
		(2)
(b)	$"300"+60\mathrm{e}^{-0.05T}=100+80\mathrm{e}^{0.05T}\Rightarrow ...\mathrm{e}^{0.05T}\pm...\mathrm{e}^{-0.05T}\pm...=0$	M1
	$4\mathrm{e}^{0.1T}-10\mathrm{e}^{0.05T}-3=0\Rightarrow \mathrm{e}^{0.05T}=\dfrac{10+\sqrt{148}}{8}$ oe	M1
	$\Rightarrow T=\dfrac{\ln(...)}{0.05}$	dM1
	$T=20.3819....=20.4$	A1
		(4)
		(6 marks)

Notes:

(a)

M1 Sets the number of guinea pigs equal to $2\times$ number of rabbits and sets $t=0$

A1 300

(b)

M1 Sets their $"300"+60\mathrm{e}^{-0.05T}=100+80\mathrm{e}^{0.05T}$ and rearranges to produce a simplified equation of the form $...\mathrm{e}^{0.05T}\pm...\mathrm{e}^{-0.05T}\pm...=0$

M1 Sets up and solves their 3TQ in $\mathrm{e}^{0.05T}$

dM1 Depends on previous M mark. Solves their $\mathrm{e}^{0.05T}=C,\quad C>0$ using lns to find value for T

A1 20.4 only

Question	Scheme	Marks
4	Possible solutions include: Solution from $R\sin(2\theta+\alpha)$ or $R\cos(2\theta-\alpha)$ Eg Attempt at $3\sin 2\theta+5\cos 2\theta = R\sin(2\theta+\alpha)$ $R=\sqrt{3^2+5^2} \quad (=\sqrt{34})$ $\alpha=\tan^{-1}\left(\frac{5}{3}\right)\Rightarrow \alpha = \ldots$ $\alpha=$ awrt 1.03 $"\sqrt{34}"\sin(2\theta\pm"1.03")=4\Rightarrow \sin(2\theta\pm"1.03")=\frac{4}{"\sqrt{34}"}$ $2\theta\pm"1.03"=\sin^{-1}\left(\frac{4}{"\sqrt{34}"}\right)\Rightarrow \theta_1=\frac{(2n+1)\pi-\sin^{-1}\left(\frac{4}{"\sqrt{34}"}\right)\mp"1.03"}{2}$ or $\theta_2=\frac{\sin^{-1}\left(\frac{4}{"\sqrt{34}"}\right)+2n\pi\mp"1.03"}{2}$ $\theta=$ awrt 0.68, 3.0, 3.8, 6.1	 M1 M1 A1 M1 dM1 A1A1
Alt1	Solution using double angle formulae, e.g $3\sin 2\theta+5\cos 2\theta=4$ $6\sin\theta\cos\theta+5\left(2\cos^2\theta-1\right)=4$ $10\cos^2\theta+6\sin\theta\cos\theta=9$ $10+6\tan\theta=9\sec^2\theta$ $10+6\tan\theta=9\left(1+\tan^2\theta\right)$	 M1 M1A1 M1

	$9\tan^2\theta - 6\tan\theta - 1 = 0$ $\tan\theta = \dfrac{6 \pm \sqrt{72}}{18}$ $\theta =$ awrt 0.68, 3.0, 3.8, 6.1	dM1 A1A1
Alt2	Solution using trigonometric identities, e.g $3\sin 2\theta + 5\cos 2\theta = 4$ $3\tan 2\theta + 5 = 4\sec 2\theta$ $9\tan^2 2\theta + 30\tan 2\theta + 25 = 16\sec^2 2\theta$ $9\tan^2 2\theta + 30\tan 2\theta + 25 = 16\left(1 + \tan^2 2\theta\right)$ $7\tan^2 2\theta - 30\tan 2\theta - 9 = 0$ $\tan 2\theta = \dfrac{30 \pm \sqrt{1152}}{14} = -0.2815, 4.567$ $\theta = \dfrac{\arctan\text{"}-0.2815\text{"}}{2}$ or $\theta = \dfrac{\arctan\text{"}4.567\text{"}}{2}$ $\theta =$ awrt 0.68, 3.0, 3.8, 6.1	 M1 M1 A1 M1 dM1 A1A1
		(7 marks)

Notes:

There are many ways to solving this question. Examples of three types of solution are given in the main scheme with the notes below. Other ways can be marked similarly.

Attempt using $R\sin(2\theta \pm \alpha)$

M1 Attempts to find R (may be given on sight of of $\sqrt{3^2+5^2}$ or $\sqrt{34}$)

M1 For sight of $\tan\alpha = \pm\dfrac{5}{3}$, $\tan\alpha = \pm\dfrac{3}{5}$. Condone $\sin\alpha = 5,\ \cos\alpha = 3 \Rightarrow \tan\alpha = \dfrac{5}{3}$.

If R is found first, accept $\sin\alpha = \pm\dfrac{5}{R}, \cos\alpha = \pm\dfrac{3}{R}$

A1 $\alpha =$ awrt 1.03. (The degree equivalent $59.0°$ is A0)

M1 Sets their $\text{"}\sqrt{34}\text{"}\sin(2\theta \pm \text{"}1.03\text{"}) = 4$ and proceeds to $\sin(2\theta \pm \text{"}1.03\text{"}) = \dfrac{4}{\text{"}\sqrt{34}\text{"}}$

M1 A correct method to find one of the solutions. As a minimum they should find $2n\pi + \sin^{-1}\left(\frac{4}{"\sqrt{34}"}\right)$ where $n = 1$ or 2 and proceeds correctly to find $\theta = ...$ (awrt 3.0 or awrt 6.1 is sufficient evidence of this). Alternatively they find $n\pi - \sin^{-1}\left(\frac{4}{"\sqrt{34}"}\right)$ where $n = 1$ or 2 and proceeds correctly to find $\theta = ...$ (awrt 0.68 or awrt 3.8 is sufficient evidence of this)

A1 Any two of $\theta =$ awrt 0.68, 3.0, 3.8, 6.1

A1 $\theta =$ awrt 0.68, 3.0, 3.8, 6.1 only

Alt1 Attempt using double angle formulae

M1 Uses both $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 2\cos^2\theta - 1$

M1 Divides by $\cos^2\theta$ to form an equation in $\tan\theta$ and $\sec\theta$

A1 Correct equation

M1 Uses $1 + \tan^2\theta = \sec^2\theta$ to form a quadratic equation in $\sec\theta$

M1 A correct method to find one of the values of θ

A1 Any two of $\theta =$ awrt 0.68, 3.0, 3.8, 6.1

A1 $\theta =$ awrt 0.68, 3.0, 3.8, 6.1 only

Alt2 Attempt using trigonometric identities

M1 Divides by $\cos 2\theta$ to form an equation in $\tan 2\theta$ and $\sec 2\theta$

M1 Attempts to square and uses the identity $1 + \tan^2 2\theta = \sec^2 2\theta$

A1 Correct equation

M1 Attempts to solve the quadratic equation to find at least one value for $\tan 2\theta$

M1 A correct method to find one of the values of θ

A1 Any two of $\theta =$ awrt 0.68, 3.0, 3.8, 6.1

A1 $\theta =$ awrt 0.68, 3.0, 3.8, 6.1 only

Question	Scheme	Marks
5(a)	$\dfrac{7}{x+3}-\dfrac{5x+22}{(x+3)(x+4)}=\dfrac{7(x+4)-5x+22}{(x+3)(x+4)}$	M1
	$=\dfrac{7x+28-5x+22}{(x+3)(x+4)}=\dfrac{2x+6}{(x+3)(x+4)}$	M1
	$=\dfrac{2(x+3)}{(x+3)(x+4)}=\dfrac{2}{x+4}$	A1
		(3)
(b)	$y=\dfrac{2}{x+4}\Rightarrow x=\ldots$	M1
	$\Rightarrow \mathrm{f}^{-1}(x)=\dfrac{2}{x}-4$	A1
	$(x\in □,)\ 0<x<2$	B1
		(3)
(c)	$\{\mathrm{ff}(x)=\}\ \dfrac{2}{\dfrac{2}{x+4}+4}=\dfrac{2}{5}$	B1
	$10(x+4)=2(2+4x+16)\Rightarrow x=\ldots$	M1
	$x=-2$	A1
		(3)
		(9 marks)

Notes:

(a)

M1 Writes both fractions with the same common denominator or writes as a single fraction

M1 Simplifies the numerator and denominator to a form of $\dfrac{\text{linear}}{\text{quadratic}}$

A1 Cancels to achieve the required answer cao

(b)

M1 Correct method to find the inverse.

A1 $\mathrm{f}^{-1}(x) = \frac{2}{x} - 4$

B1 $(x \in \mathbb{R},)\ 0 < x < 2$

(c)

B1 Sight of the correct starting equation. This may be implied by later working.

Note an alternative method via $\mathrm{f}(x) = \mathrm{f}^{-1}\left(\frac{2}{5}\right) \Rightarrow \frac{2}{x+4} = 1$

M1 A correct method to find x.

A1 $x = -2$

Question	Scheme	Marks
6(a)	At P $x>\pi \Rightarrow y=-\sin x+1$	
	$\frac{dy}{dx}=-\cos x=-\frac{1}{2}\Rightarrow x=...$	M1
	$a=\frac{5\pi}{3}$	A1
	y coordinate of $P=\left\|\sin"\frac{5\pi}{3}"\right\|+1=...$	dM1
	$b=\frac{\sqrt{3}}{2}+1$	A1
		(4)
(b)	$m=\frac{"\frac{\sqrt{3}}{2}+1"-1}{"\frac{5\pi}{3}"-0}$ or $m=\frac{"\frac{\sqrt{3}}{2}+1"-1}{"\frac{5\pi}{3}"-\pi}$	M1
	$m=\frac{"\frac{\sqrt{3}}{2}+1"-1}{"\frac{5\pi}{3}"-0}$ and $m=\frac{"\frac{\sqrt{3}}{2}+1"-1}{"\frac{5\pi}{3}"-\pi}$	M1
	$\frac{3\sqrt{3}}{10\pi}<m<\frac{3\sqrt{3}}{4\pi}$	A1
		(3)
		(7 marks)

Notes:

(a)

M1 Differentiates $\pm\sin x$ to achieve $\pm\cos x=\pm\frac{1}{2}$ and proceeds to achieve an angle for x.

May come from symmetry or periodicity of $\sin x$

A1 $a=\frac{5\pi}{3}$

dM1 Substitutes their value of a for x to find b.

A1 $b=\frac{\sqrt{3}}{2}+1$

(b)

M1 Attempts to find the gradient of the line between their P and either $(0,1)$ or $(\pi,1)$

M1 Attempts to find the gradient of the line between their P and both $(0,1)$ and $(\pi,1)$

A1 $\frac{3\sqrt{3}}{10\pi}<m<\frac{3\sqrt{3}}{4\pi}$ or $\frac{3\sqrt{3}}{10\pi}<m$ AND $m<\frac{3\sqrt{3}}{4\pi}$ or $\left(\frac{3\sqrt{3}}{10\pi},\frac{3\sqrt{3}}{4\pi}\right)$ or exact equivalent.

DO NOT accept $\frac{3\sqrt{3}}{10\pi}<m$ OR $m<\frac{3\sqrt{3}}{4\pi}$

Question	Scheme	Marks
7(a)	$\frac{dy}{dx} = ...e^{2\sqrt{3}x}\cos 2x \pm ...e^{2\sqrt{3}x}\sin 2x$	M1
	$\frac{dy}{dx} = 2\sqrt{3}e^{2\sqrt{3}x}\cos 2x - 2e^{2\sqrt{3}x}\sin 2x$	A1
		(2)
(b)	$"2\sqrt{3}e^{2\sqrt{3}x}\cos 2x - 2e^{2\sqrt{3}x}\sin 2x" = 0 \Rightarrow$ $e^{2\sqrt{3}x}(...\cos 2x - ...\sin 2x) = 0$	M1
	$...\cos 2x - ...\sin 2x = 0 \Rightarrow \tan 2x = "\sqrt{3}"$	M1
	$2x = \tan^{-1}("\sqrt{3}") \Rightarrow x = ...$	M1
	$x = \frac{\pi}{6}, \frac{2\pi}{3}$	A1
	$y = e^{\left(2\sqrt{3}\times"\frac{\pi}{6}"\right)}\cos\left(2\times"\frac{\pi}{6}"\right) = ...$	M1
	$\left(\frac{\pi}{6}, \frac{1}{2}e^{\frac{\sqrt{3}\pi}{3}}\right)$ and $\left(\frac{2\pi}{3}, -\frac{1}{2}e^{\frac{4\sqrt{3}\pi}{3}}\right)$	A1
		(6)
		(8 marks)

Notes:

(a)

M1 Attempts to use the product rule to achieve an expression of the form $\frac{dy}{dx} = ...e^{2\sqrt{3}x}\cos 2x \pm ...e^{2\sqrt{3}x}\sin 2x$

A1 Correct unsimplified expression.

(b)

M1 Sets their $\frac{dy}{dx} = 0$ and attempts to factorise by taking out the exponential term.

M1 Attempts to solve their factorised expression/rearranges their trigonometric solution to $\tan 2x = ...$

M1 Attempts to find one angle from their trigonometric equation.

A1 Both x coordinates correct.

M1 Attempts to find the corresponding y coordinate for one of their x coordinates.

A1 Both coordinates correct (accept any exact equivalent forms).

Question	Scheme	Marks
8(a)	$P = ab^{-t}$ $\log_{10} P = \log_{10} ab^{-t} \Rightarrow \log_{10} P = \log_{10} a + \log_{10} b^{-t}$ $\Rightarrow \log_{10} P = \log_{10} a - t\log_{10} b$ *	B1*
		(1)
(b)	$\log_{10} a = 1.6 \Rightarrow a =$ awrt 39.81	B1
	Using $(10,1.4)$, $t=10$, $\log_{10} P = 1.4$ $\log_{10} P = \log_{10} a - 10\log_{10} b \Rightarrow 1.4 = 1.6 - 10\log_{10} b$	M1
	$1.4 = 1.6 - 10\log_{10} b \Rightarrow \log_{10} b = "0.02" \Rightarrow b = 10^{"0.02"}$	M1
	$b =$ awrt 1.047	A1
		(4)
(c)	$\frac{\mathrm{d}P}{\mathrm{d}t} = -a\ln b \times b^{-t}$	M1
	$\frac{\mathrm{d}P}{\mathrm{d}t} = -"39.81" \times \ln("1.047") \times "1.047"^{-8}$	M1
	$\frac{\mathrm{d}P}{\mathrm{d}t} = -1.26836... \Rightarrow$ decrease of awrt 1270 people per year	A1
		(3)
		(8 marks)
Alt1(b)	$\log_{10} a = 1.6 \Rightarrow a =$ awrt 39.81	B1
	$-\log_{10} b = \frac{1.4-1.6}{10-0} = ...$	M1
	$\log_{10} b = "0.02" \Rightarrow b = 10^{"0.02"}$	M1
	$b =$ awrt 1.047	A1

Notes:

(a)

B1* Must see **taking lns of both sides, using the addition rule** to write as two separate logs and **using the power rule** to achieve the given answer with no errors.

(b)

B1 awrt 39.81

M1 Uses $(10,1.4)$ by substituting into the equation of the line. Alternatively attempts to calculate the gradient of the line using $(0,1.6)$ and $(10,1.4)$

M1 Rearranges to make $\log_{10} b$ the subject and carries out correct log work to find a value for *b*. Alternatively sets their gradient equal to $\log_{10} b$ and proceeds correctly to find *b*.

A1 awrt 1.047

(c)

M1 Differentiates to find $\frac{\mathrm{d}P}{\mathrm{d}t}$ in the form $\ldots \ln b \times b^{-t}$ with their values from (b)

M1 Substitutes $t=8$ into their $\frac{\mathrm{d}P}{\mathrm{d}t}$ (it must be a changed expression i.e it cannot be the original equation).

A1 decrease of awrt 1270 people per year. Allow -1270 people per year. Must include units.

Question	Scheme	Marks
9(a)	$x=\dfrac{(\sin y-\cos y)(\sin y+\cos y)}{(\cos y+\sin y)(\cos y+\sin y)}\Rightarrow x=\dfrac{\sin^2 y-\cos^2 y}{\cos^2 y+2\sin y\cos y+\sin^2 y}$	M1
	$\Rightarrow x=\dfrac{-\cos 2y}{1+\sin 2y}$ *	M1A1*
		(3)
(b)	Quotient rule:	
	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{(1+\sin 2y)\times 2\sin 2y-(-\cos 2y)\times 2\cos 2y}{(1+\sin 2y)^2}$	M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{2\sin 2y+2\sin^2 2y+2\cos^2 y}{(1+\sin 2y)^2}=\dfrac{2\sin 2y+2}{(1+\sin 2y)^2}$	
	$\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{2}{1+\sin 2y}$ *	A1*
		(2)
(c)	$\dfrac{1+\sin 2y}{2}=\dfrac{1}{4}\Rightarrow \sin 2y=\ldots$	M1
	$\sin 2y=\text{"}\dfrac{-1}{2}\text{"}\Rightarrow y=\dfrac{1}{2}\sin^{-1}\left(\text{"}\dfrac{-1}{2}\text{"}\right)$	M1
	$y=-\dfrac{\pi}{12}$	A1
	$x=\dfrac{-\cos\left(\text{"}-\dfrac{\pi}{6}\text{"}\right)}{1+\sin\left(\text{"}-\dfrac{\pi}{6}\text{"}\right)}\Rightarrow x=\ldots$	M1
	$x=-\sqrt{3}$	A1
		(5)
		(10 marks)
Alt1(b)	Product rule $x=-\cos 2y(1+\sin 2y)^{-1}$	

	$\frac{dx}{dy} = 2\sin 2y(1+\sin 2y)^{-1} + (-\cos 2y)(-1)(2\cos 2y)(1+\sin 2y)^{-2}$ $\frac{dx}{dy} = \frac{2\sin 2y(1+\sin 2y)}{(1+\sin 2y)^2} + \frac{2\cos^2 2y}{(1+\sin 2y)^2}$ $\Rightarrow \frac{dx}{dy} = \frac{2\sin 2y + 2\sin^2 2y + 2\cos^2 2y}{(1+\sin 2y)^2} = \frac{2\sin 2y + 2}{(1+\sin 2y)^2}$ $\Rightarrow \frac{dx}{dy} = \frac{2}{1+\sin 2y}$ *	M1 A1*

Notes:

(a)

M1 Attempts to multiply by $\frac{\sin y + \cos y}{\sin y + \cos y}$ and attempts to use **either**

$\cos^2 y - \sin^2 y = \cos 2y$ **or** $\sin 2y = 2\sin y \cos y$

Allow one error in multiplying out the brackets

M1 Attempts to multiply by $\frac{\sin y + \cos y}{\sin y + \cos y}$ and attempts to use **both**

$\cos^2 y - \sin^2 y = \cos 2y$ **and** $\sin 2y = 2\sin y \cos y$

Allow one error in multiplying out the brackets

A1* Fully correct method with no errors or omissions including brackets. The identities should be stated or seen to be correctly substituted.

Note, if working in reverse, then the M1 is scored for replacing 1 by $\sin^2 y + \cos^2 y$

and attempting to use **either** $\cos 2y = \cos^2 y - \sin^2 y$ **or** $\sin 2y = 2\sin y \cos y$

(b)

M1 Attempts to correctly apply the quotient rule. If the formula is not quoted then look for an expression of the form

$$\frac{dx}{dy} = \frac{(1+\sin 2y)\times \ldots \sin 2y \pm \cos 2y \times \ldots \cos 2y}{(1+\sin 2y)^2} \text{ or equivalent}$$

In the alternative method they attempt the product rule. If the formula is not quoted then look for an expression of the form

$$\frac{dx}{dy} = ...\sin 2y(1+\sin 2y)^{-1} \pm ...\cos^2 2y(1+\sin 2y)^{-2} \text{ or equivalent}$$

Condone invisible brackets for this mark

A1* Correctly proceeds with no errors to the given answer including bracket errors.

(c)

M1 Sets $\frac{1+\sin 2y}{2} = \frac{1}{4}$ and rearranges to $\sin 2y = ...$ Condone slips in the rearrangement for this mark.

M1 Proceeds correctly to finding a value for y using their value for $\sin 2y$

A1 $y = -\frac{\pi}{12}$ Ignore any sight of $y = \frac{7\pi}{12}$

M1 Substitutes their value for y into $x = \frac{-\cos 2y}{1+\sin 2y}$ and attempts to find a value for x. The expression may be unsimplified but the trigonometric functions must be evaluated and be exact.

A1 $x = -\sqrt{3}$ cao They do not have to write $\left(-\sqrt{3}, -\frac{\pi}{12}\right)$ but withhold this mark if they have more than one pair of coordinates eg $\left(\sqrt{3}, \frac{7\pi}{12}\right)$

Question	Scheme	Marks
10(a)	$e^{2\alpha-3}-\frac{4}{3\alpha}=0\Rightarrow e^{2\alpha-3}=...\Rightarrow 2\alpha-3=\ln\left(\text{"}\frac{4}{3\alpha}\text{"}\right)$	M1
	$\alpha=\frac{1}{2}\left(\ln\left(\frac{4}{3\alpha}\right)+3\right)$ *	A1*
		(2)
(b)	$x_1=\frac{1}{2}\left(\ln\left(\frac{4}{3\times 2}\right)+3\right)=....$	M1
	$x_1=$ awrt 1.2973 $x_5=$ awrt 1.4537	A1
		(2)
(c)	Using $f(x)=e^{2x-3}-\frac{4}{3x}$	
	$f(1.4555)=-0.0012199...<0$ $f(1.4565)=0.0012405...>0$	M1
	As f(x) is continuous on the required interval and there is a change of sign $\Rightarrow$ $\underline{\alpha=1.456}$ (3 decimal places)	A1
		(2)
(d)	$\int_{-4}^{-2}\left(e^{2x-3}-\frac{4}{3x}\right)dx$	M1
	$\int\left(e^{2x-3}-\frac{4}{3x}\right)dx=...e^{2x-3}\pm...\ln\lvert x\rvert\quad(+C)$	M1
	$\int\left(e^{2x-3}-\frac{4}{3x}\right)dx=\frac{1}{2}e^{2x-3}-\frac{4}{3}\ln\lvert x\rvert$	A1
	$=\left[\frac{1}{2}e^{2x-3}-\frac{4}{3}\ln\lvert x\rvert\right]_{-4}^{-2}=\left(\frac{1}{2}e^{-7}-\frac{4}{3}\ln\lvert -2\rvert\right)-\left(\frac{1}{2}e^{-11}-\frac{4}{3}\ln\lvert -4\rvert\right)$	M1
	$=\frac{1}{2}e^{-7}-\frac{1}{2}e^{-11}+\frac{4}{3}\ln 2$	A1
		(5)
		(11 marks)

Notes:

(a)

M1 Sets $y = 0$, rearranges to $e^{2\alpha-3} = ...$ and takes lns of both sides. May still be in terms of x

A1* Achieves required form with no errors including brackets.

(b)

M1 Substitutes $x = 2$ into the given iteration formula and proceeds to finding x_1. May be implied by awrt 1.30

A1 $x_1 =$ awrt 1.2973 and $x_5 =$ awrt 1.4537 only

(c)

M1 Chooses a suitable interval eg $(1.4555, 1.4565)$ and substitutes into the equation of the curve to find corresponding y values. Alternatively they may substitute into $\frac{1}{2}\left(\ln\left(\frac{4}{3x}\right)+3\right)-x=0$ (values are 6.66×10^{-4} and -6.77×10^{-4} to 3sf)

A1 Correct values for their interval (may be rounded to 2 significant figures or truncated) Minimal conclusion stating that as the function is continuous on the required interval and there has been a change of sign this implies $\underline{\alpha = 1.456}$ to 3 decimal places.

(d)

M1 A correct strategy to find the exact area of the shaded region. Sight of $\int_{-4}^{-2}\left(e^{2x-3}-\frac{4}{3x}\right)dx$ is sufficient or it may be implied by an attempt to integrate and substitute in the limits either way round.

M1 Attempts to integrate to a form $...e^{2x-3} \pm ...\ln|x|$ or $...e^{2x-3} \pm ...\ln|3x|$

A1 $\frac{1}{2}e^{2x-3}-\frac{4}{3}\ln|x|$ or exact equivalent. Note $\ln|3x|$ is acceptable. Ignore (+C)

M1 Substitutes -4 and -2 into their changed function and subtracts either way round

A1 $\frac{1}{2}e^{-7}-\frac{1}{2}e^{-11}+\frac{4}{3}\ln 2$ or exact simplified equivalent